

Uncertainty, Stochastics & Sensitivity Analysis

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Types of Sensitivity Analyses

- Variables involved
 - One-way
 - Multi-way
- Type of component being varied
 - Parameter sensitivity analysis: Parameter values
 - Structural sensitivity analysis: Examine effects of model *structure* on results
- Type of variation
 - Single alternative values
 - Monte Carlo analyses: Draws from probability distributions (many types of variations)
- Frequency of variation
 - Static (parameter retains value all through simulation)
 - Ongoing change: Stochastic process
 - Accomplished via Monte-Carlo analyses
 - Key for DES & ABM

Model Uncertainty

- Here, we are frequently examining the impact of changing
 - Our assumptions about “how the system works”
 - Our decision of how to abstract the system behaviour
- Structural sensitivity analyses
 - Vary structure of model & see impact on
 - Results
 - Tradeoffs between choices
 - Frequently recalibrate the model in this process
- Here, we are considering uncertainty about how the current state is mapped to the next state

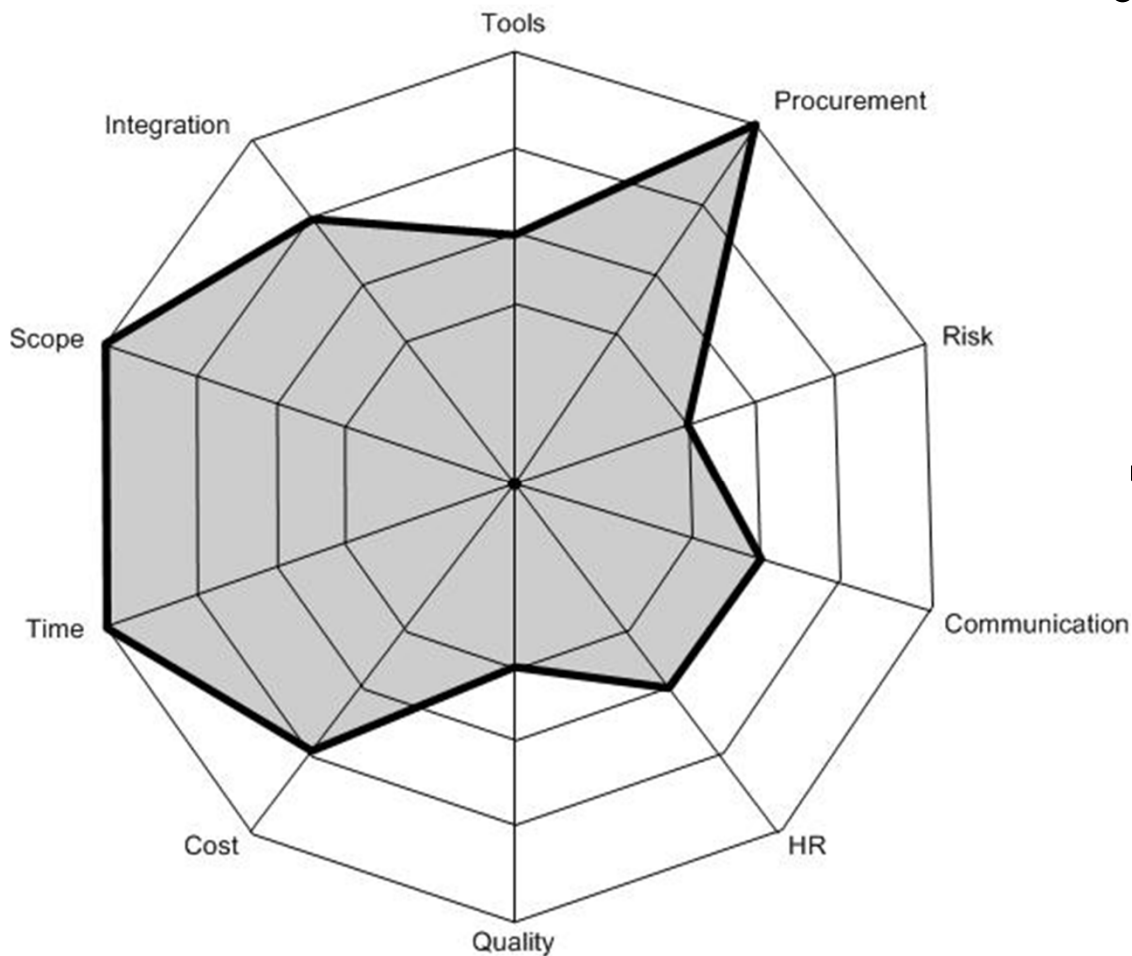
Predictor-Corrector Methods: Dealing with an Incomplete Model

- Some approaches (e.g. Kalman filter, Particle Filter) are motivated by awareness that models are incomplete
- Such approaches try to adjust model state estimates on an ongoing basis,
 - Given uncertainty about model predictions
 - New observations
- Assumption here is that the error in the model is defined by some probability distribution

Static Uncertainty Sensitivity Analyses

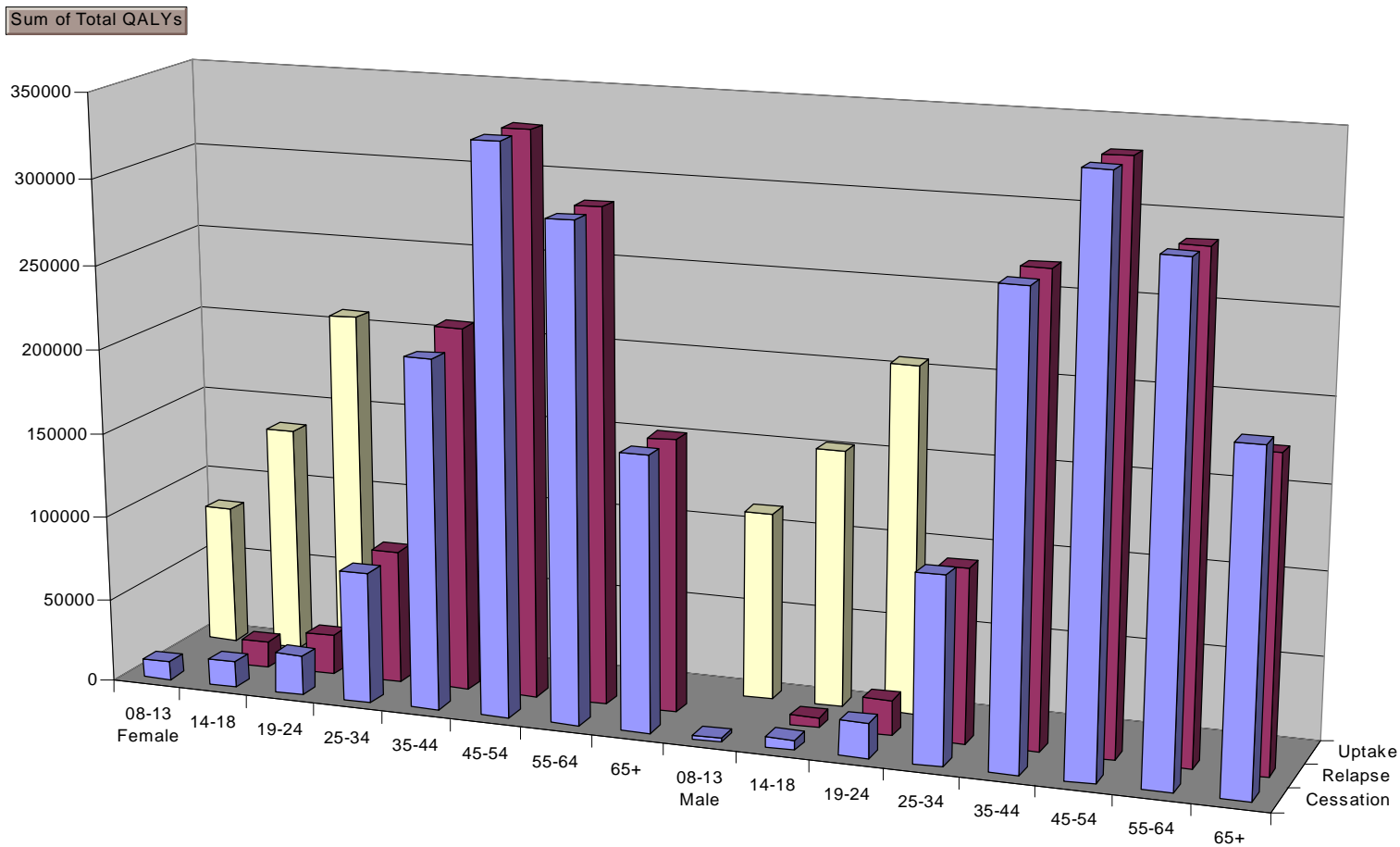
- In variation, one can seek to investigate different
 - Assumptions
 - Policies
- Same relative or absolute uncertainty in different parameters may have hugely different effect on outcomes or decisions
- Help identify parameters/initial states that strongly affect
 - Key model results
 - Choice between policies
- We place more emphasis in parameter estimation & interventions into parameters exhibiting high sensitivity

Spider Diagram



- Each axis represents a % change in a particular parameter
 - This proportional change is identical for the different parameters
- The distance assumed by the curve along that axis represents the magnitude of response to that change
 - Note that these sensitivities will depend on the state of system!

Systematic Examination of Policies



Tengs, Osgood, Lin

Add New “Parameters Variation” Experiment

New Experiment

Experiment
Select an experiment type, specify a name and choose a root (top-level) active object.

Name:

Main Active Object Class (root):

Experiment Type:

- Simulation
- Optimization
- Parameters Variation
- Compare Runs (available in Professional edition)
- Monte Carlo (available in Professional edition)
- Sensitivity Analysis (available in Professional edition)
- Calibration (available in Professional edition)
- Custom (available in Professional edition)

Performs multiple model runs varying one or more parameters, optionally using replications.
You can later on add arbitrary UI to this experiment.

Copy model time settings from :

< Back Next > Finish Cancel

Setting Ranges for Parameter Variation

The screenshot displays the AnyLogic Advanced software interface, specifically the 'ParametersVariation' window. The main workspace is a grid where a parameter variation experiment is being configured. The 'ParametersVariation - Parameter Variation Experiment' window is open, showing the following settings:

- Name:** ParametersVariation
- Main active object class (root):** Main
- Random number generation:** Random seed (unique simulation runs), Fixed seed (reproducible simulation runs) with Seed Value: 1
- Parameters:** Varied in range, Freeform, Number of runs: 10

The 'Parameters' table is as follows:

Parameter	Type	Value
AverageI...uration	range	0, 10, 1
ContactRate	fixed	1.0
Infection...bability	range	0.8
AreaSide	fixed	100
TotalPopulation	fixed	10000

The interface also shows a Project tree on the left, a Palette on the right, and a Properties/Console window at the bottom.

Sensitivity Exploration in AnyLogic

The screenshot displays the AnyLogic Advanced software interface. The main window shows a simulation titled "Agent Based SIR Model - Monte Carlo Simulation". A button labeled "Run 100 replicat..." is visible. The simulation results are visualized as a 2D histogram with a grid, showing the distribution of simulation outcomes. The y-axis ranges from 1,500 to 4,000. The histogram shows a complex, multi-modal distribution of data points.

The bottom panel, titled "MonteCarlo2DHistogram - Parameter Variation Experiment", contains the following configuration details:

- Name: MonteCarlo2DHistogram
- Main active object class (root): Main
- Random number generation: Random seed (unique simulation runs), Fixed seed (reproducible simulation runs) Seed Value: 1
- Parameters: Varied in range, Freeform Number of runs: 100

Parameter	Type	Value		
		Min	Max	Step
AverageI...uration	range	0	10	1
ContactRate	fixed	1.0		
Infection...bability	fixed	0.8		
AreaSide	fixed	100		
TotalPopulation	fixed	10000		

The right sidebar shows a palette of model elements including Parameter, Flow Aux Variable, Stock Variable, Event, Dynamic Event, Plain Variable, Collection Variable, Function, Table Function, Port, Connector, Entry Point, State, Transition, Initial State Pointer, Branch, History State, Final State, and Environment. The bottom right corner features buttons for Action, Analysis, Presentation, Connectivity, Enterprise Library, and More Libraries...

Sensitivity Analyses in Vensim

Sensitivity Control. Edit the filename to save changes to a different control file

Filename:

Number of simulations: Noise Seed:

Display warning messages

Multivariate Univariate
 Latin Hypercube Latin Grid
 File

Currently active parameters (drag to reorder)
Noise Seed=RANDOM_UNIFORM(0,10000)

Distribution

Parameter	Minimum Value	Maximum Value	Increment			
Mean Time to Recover	<input type="text" value="0"/>	<input type="text" value="10"/>	<input type="text" value="1"/>			

Sensitivity in Initial States

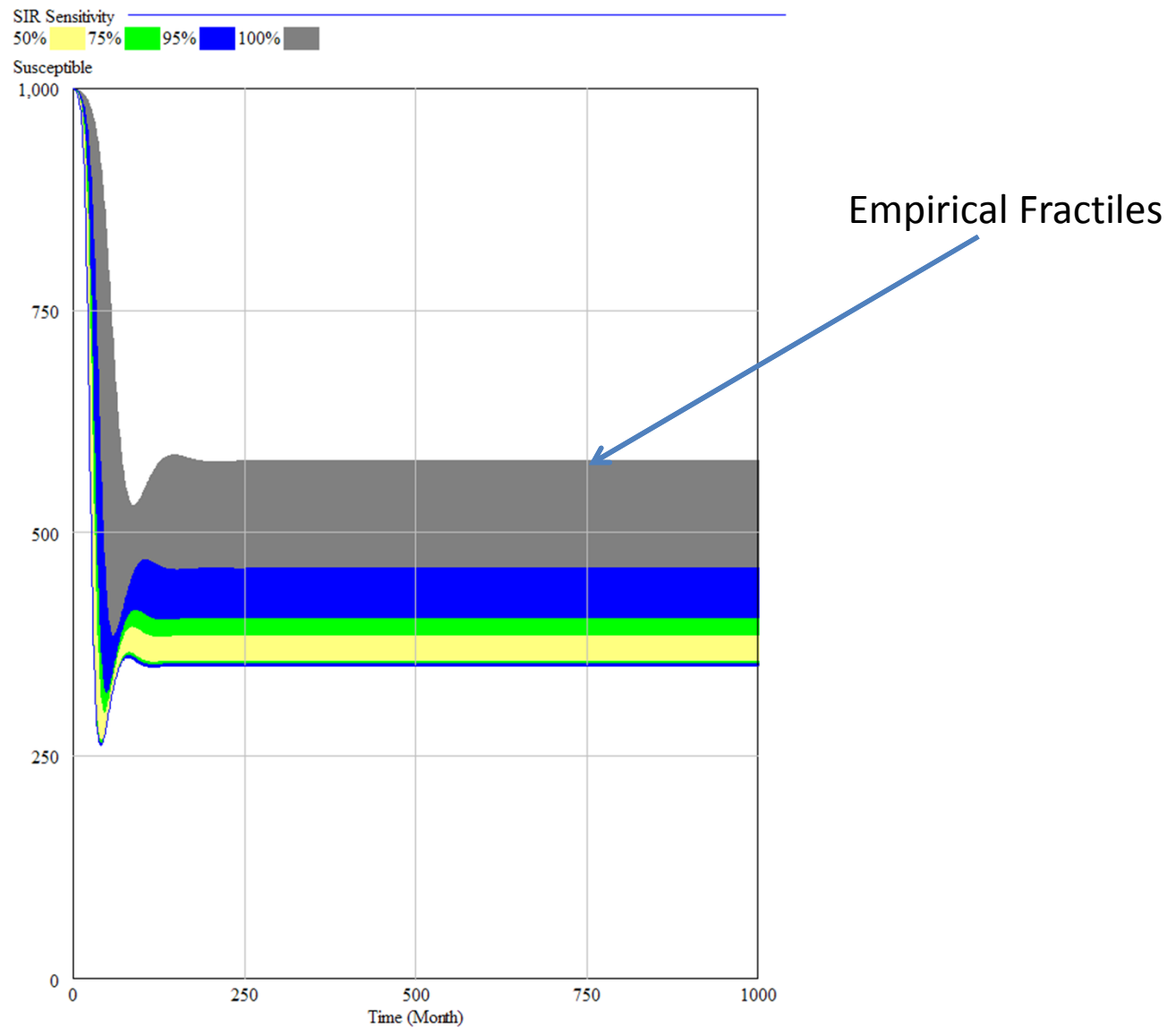
- Frequently we don't know the exact state of the system at a certain point in time
- A very useful type of sensitivity analysis is to vary the initial model state
- In Vensim, this can be accomplished by
 - Indicating a parameter name within the “initial value” area for a stock
 - Varying the parameter value
- In an agent-based model, state has far larger dimensionality
 - Can modify different numbers of people with characteristic, location of people with characteristic, etc.

Imposing a Probability Distribution

Monte Carlo Analysis

- We feed in probability distributions to reflect our uncertainty about one or more parameters
- The model is run many, many times (realizations)
 - For each realization, the model uses a different draw from those probability distribution
- What emerges is resulting probability distribution for model outputs

Example Resulting Distribution

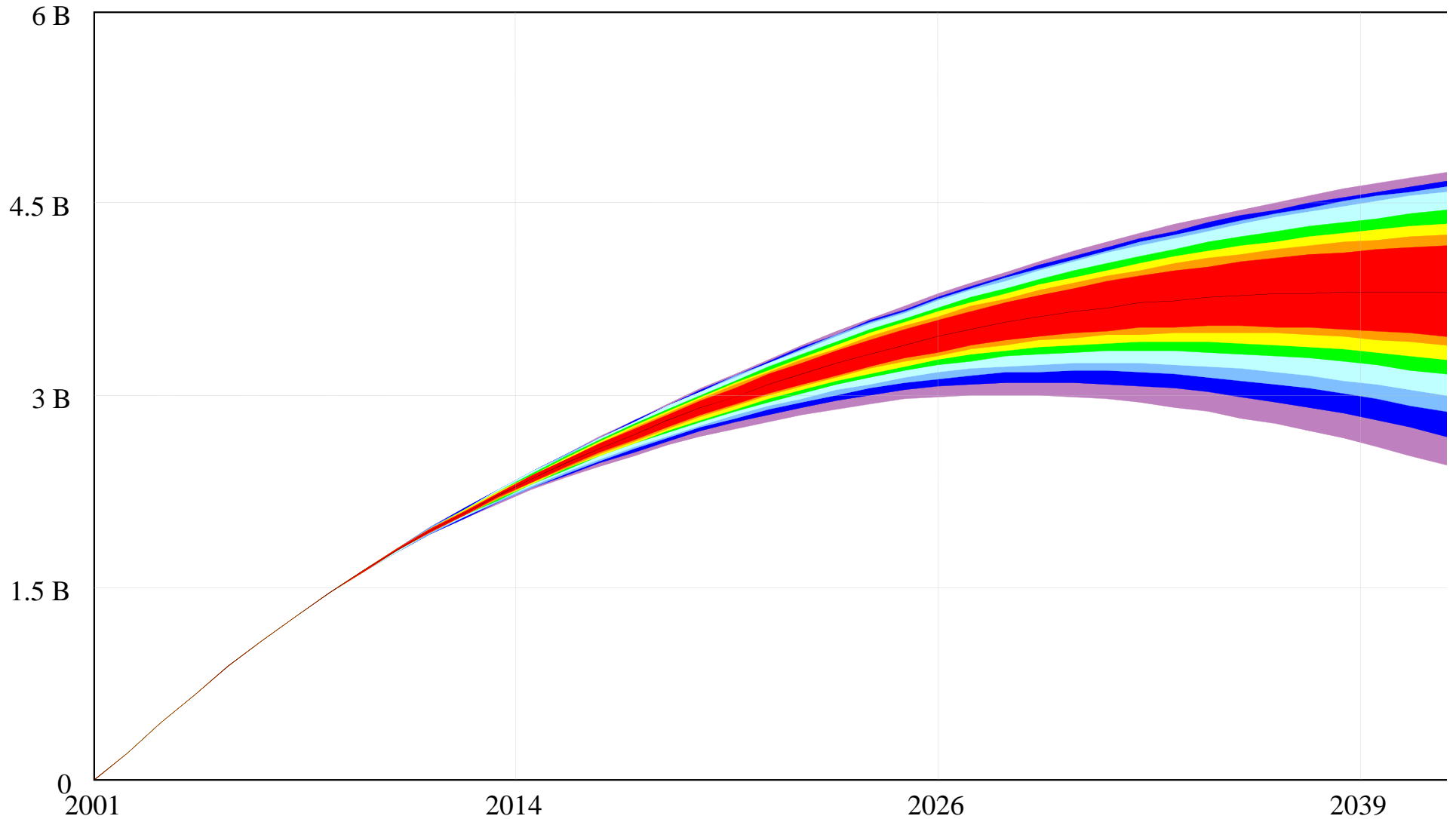


Static Uncertainty

Impact on cost of uncertainty regarding mortality and medical costs



Incremental Costs



Multi-Way Sensitivity Analyses

- When examining the results of changing multiple variables, need to consider how multiple variables vary together
- If this covariation reflects dependence on some underlying factor, may be able to simulate uncertainty in underlying factor

Performing Monte Carlo Sensitivity Analyses in Vensim

- Need to specify three things
 - The parameters to vary
 - How to vary those parameters
 - Which model variables to save away

How & What Parameters to Vary

Sensitivity Control. Edit the filename to save changes to a different control file

Filename:

Number of simulations Noise Seed Multivariate Univariate
 Latin Hypercube Latin Grid
 File

Display warning messages

Currently active parameters (drag to reorder)

Annual Birth and Death Rate=RANDOM_NORMAL(0,.05,.02,.01)

Distribution

Parameter	Value	Minimum Value	Maximum Value				
Per Contact Risk of Infection	0.05	.02	.1				

Model Values to Save Away

Savelist Control. Edit the filename to save changes to a different control file

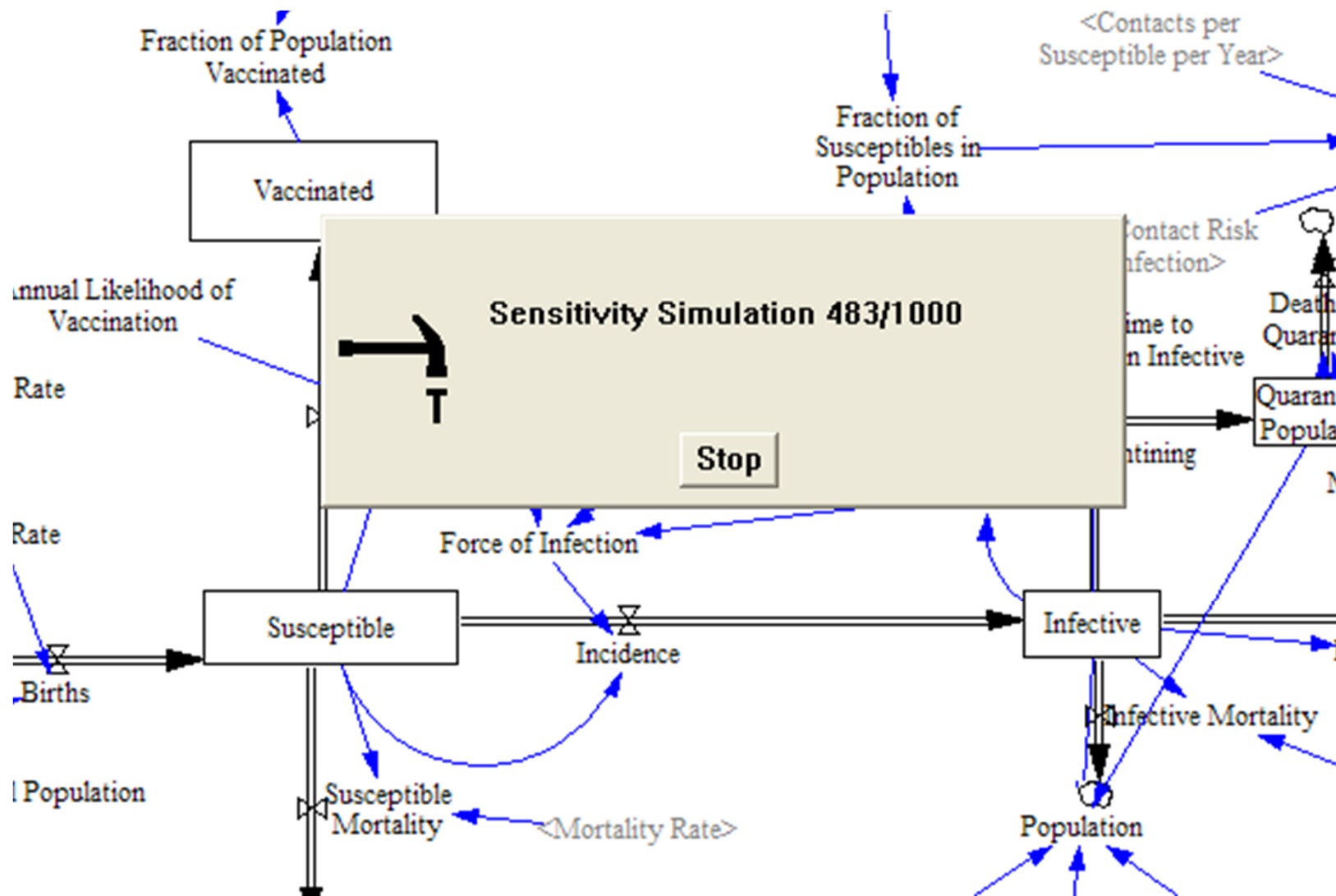
Filename:

List of Variables to be Saved (drag to reorder)

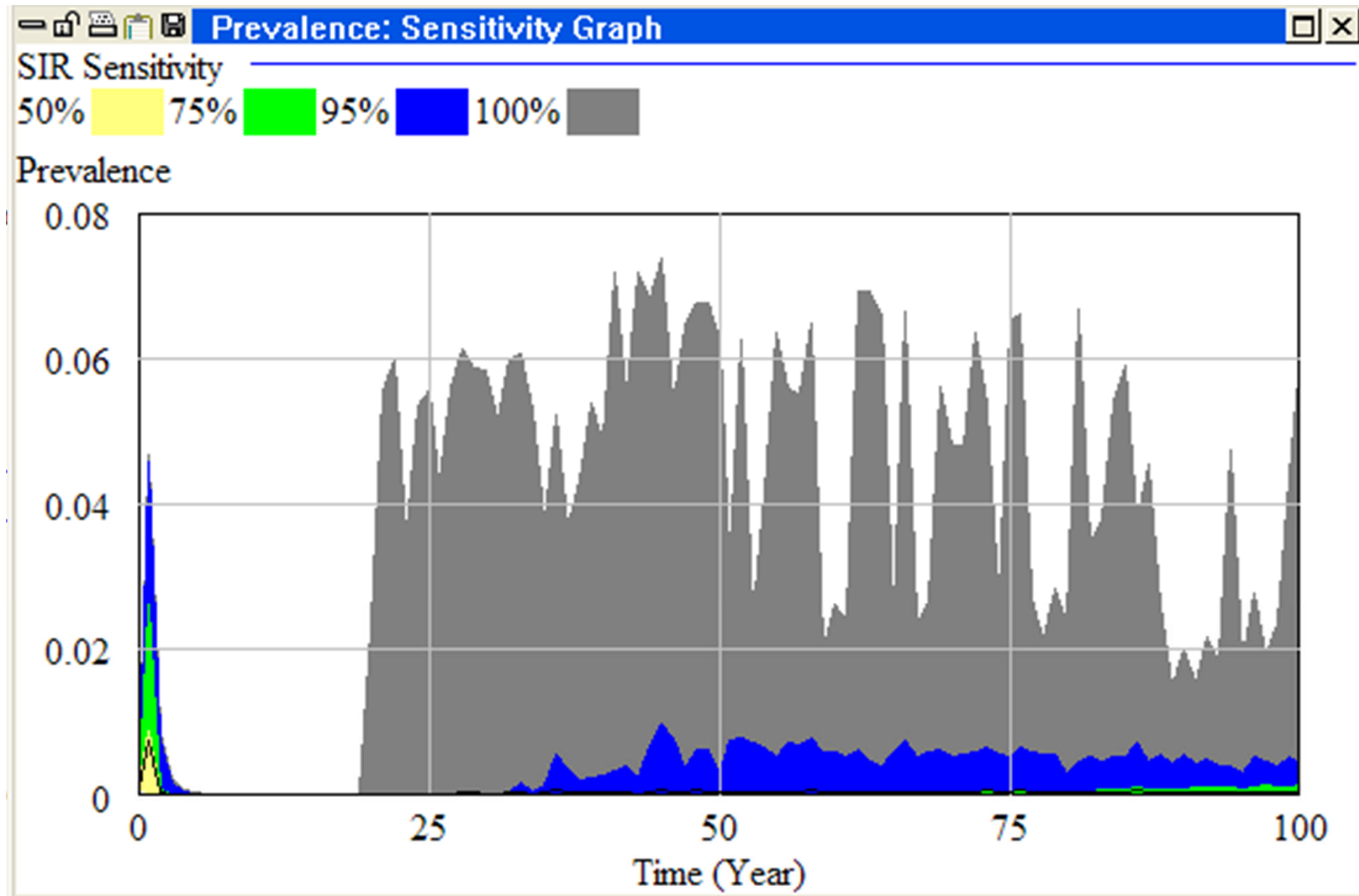
- Susceptible
- Infective
- Recovered
- Incidence
- Recovery
- Fraction of Susceptibles in Population
- Prevalence

For subscripted variables leave the subscripts off to save all elements.

Monte Carlo Analyses

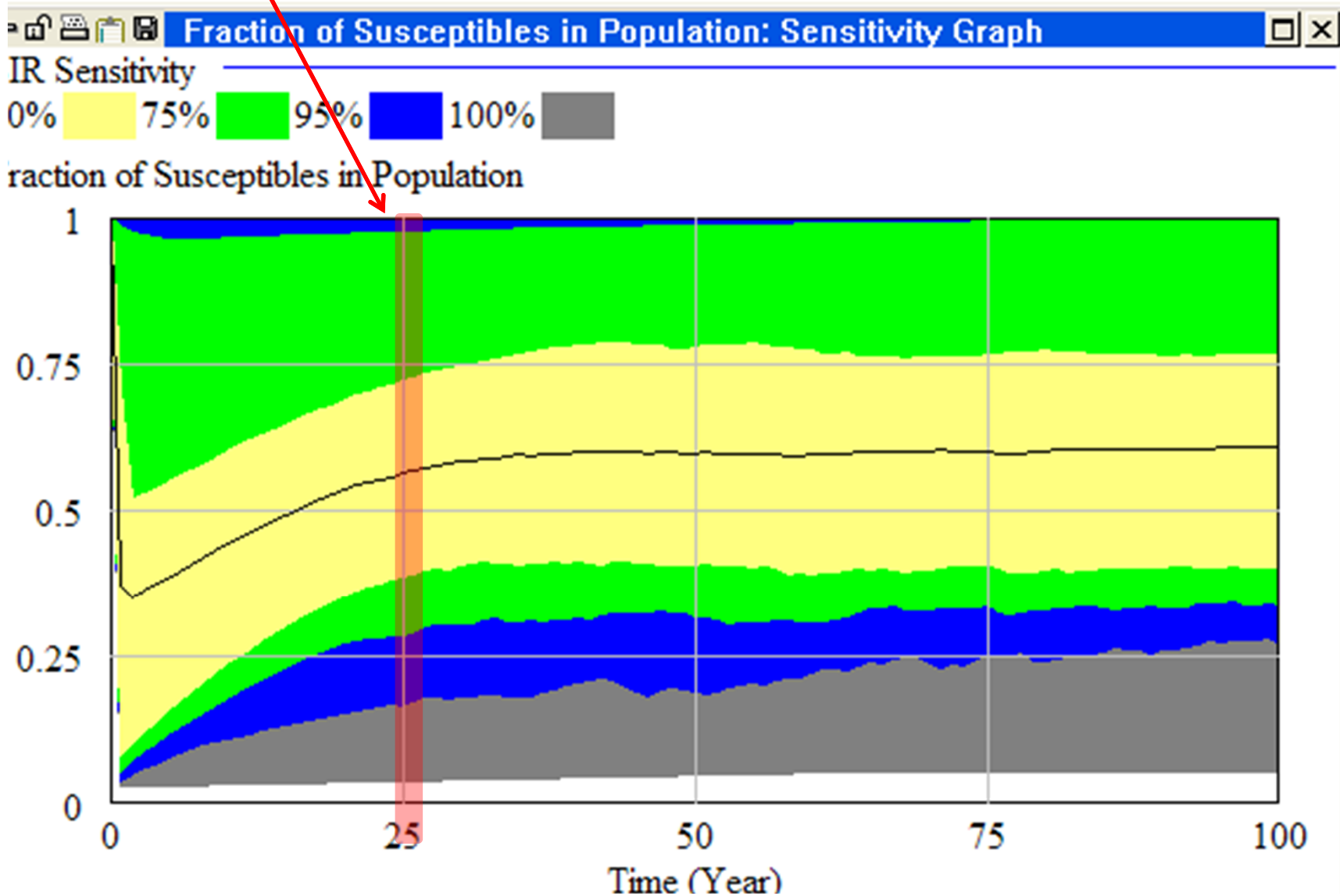


Sensitivity Results (Prevalence)



An observation at this point in time would produce a histogram (approximating a distribution) for fraction of susceptibles

Sensitivity Results (Fraction of Susceptibles)



Monte Carlo Analyses in AnyLogic

- When running Monte Carlo analysis, we'd like to summarize the results of multiple runs
- One option would be to display each trajectory over time; downside: quickly gets messy
- AnyLogic's solution
 - Accumulate data regarding how many trajectories fall within given areas of value for a given interval of time using a "Histogram2D Data"
 - Display the Histogram2D Chart



Hands on Model Use Ahead



Load Sample Model:

SIR Agent Based Calibration

(Via “Sample Models” under “Help” Menu)

2D Histogram Data

The screenshot displays the AnyLogic Advanced software interface. The main workspace shows a 2D histogram titled "Agent Based SIR M" with a "Run 100 replicat..." button. The histogram data is labeled "dataInfectious2D".

The configuration panel for "dataInfectious2D - Histogram 2D Data" is visible, showing the following settings:

- Name: dataInfectious2D
- Show Name
- Ignore
- Public
- Show At Runtime
- Horizontal axis value: []
- Vertical axis value: []
- Horizontal intervals: 80
- Range, from: 0 to: 200
- Vertical intervals: 40
- Range, from: 0 to: 4000
- Envelopes: 0.25, 0.5, 0.75
- Do not update automatically
- Auto update after every iteration

The interface also includes a Project tree on the left, a Palette on the right, and a Properties/Console area at the bottom.

Important Distinction (Declining Order of Aggregation)

- Experiment
 - Collection of simulation
- Simulation
 - Collection of replications that can yield findings across set of replications (e.g. mean value)
- Replication
 - One run of the model

Flexibility Typically Ignored

- In most AnyLogic models, an Experiment is composed of a single Simulation, which is composed of a single Replication
- In most AnyLogic models which run “ensembles” of realizations, a simulation is composed of only a single realization

Accumulating the Histogram2D dataset from other datasets

The screenshot displays the AnyLogic Advanced software interface, titled "AnyLogic Advanced [EDUCATIONAL USE ONLY]". The main workspace shows a grid with a data point labeled "dataInfectious2D". A panel titled "Agent Based SIR M" is visible, featuring a "Run 100 replicat..." button and a small 2D histogram plot with axes ranging from 3,500 to 4,000.

The left sidebar contains a project tree with the following structure:

- SIR Agent Based Calibration*
 - Main
 - Parameters
 - Plain Variables
 - Environments
 - Embedded Objects
 - Analysis Data
 - dsInfectious
 - Presentation
 - Person
 - Calibration: Main
 - MonteCarlo2DHistogram: Main
 - Analysis Data
 - dataInfectious2D
 - Presentation
 - ParametersVariation: Main
- Influenza*
 - Family
 - Main
 - Parameters
 - Plain Variables
 - Embedded Objects

The bottom panel, titled "MonteCarlo2DHistogram - Parameter Variation Experiment", shows the following configuration:

- General**: Additional Class Code: []
- Advanced**: Initial Experiment Setup: []
- Before Each Experiment Run:** `dataInfectious2D.reset();`
- Before Simulation Run:** []
- After Simulation Run:** `dataInfectious2D.add(root.dsInfectious);`
- After Iteration Code:** []
- Tolerances:** []

The right sidebar contains a "Palette" with various analysis and presentation options:

- Model
- Action
- Analysis
 - Data Set
 - Statistics
 - Histogram Data
 - Histogram2D Data
 - Bar Chart
 - Stack Chart
 - Pie Chart
 - Plot
 - Time Plot
 - Time Stack Chart
 - Time Color Chart
 - Histogram
 - Histogram2D
- Presentation
- Connectivity
- Enterprise Library
- More Libraries...

Monte Carlo Sensitivity Analyses in AnyLogic

The screenshot displays the AnyLogic Advanced software interface for a Monte Carlo simulation. The main window is titled "Agent Based SIR Model - Monte Carlo Simulation" and features a 2D histogram showing the distribution of infectious population over time. The histogram has a vertical axis ranging from 1,500 to 4,000 and a horizontal axis representing time. A button labeled "Run 100 replicat..." is visible above the histogram. The software interface includes a menu bar (File, Edit, View, Model, Window, Help), a toolbar, and a project tree on the left. The bottom panel shows the "chart - Histogram2D" properties, including the title "Dynamics of Infectious Populatic" and the data source "dataInfectious2D".

Agent Based SIR Model - Monte Carlo Simulation

Run 100 replicat...

technologies
AnyLogic and this model is (c) XJ Technologies, www.anylogic.com. All rights reserved.

chart - Histogram2D

General Name: chart Show Name Ignore Public

Advanced

Dynamic Title: Dynamics of Infectious Populatic

Appearance Histogram: dataInfectious2D Color: slateGray

Add Histogram Data

Show envelopes Show bins

Do not update automatically

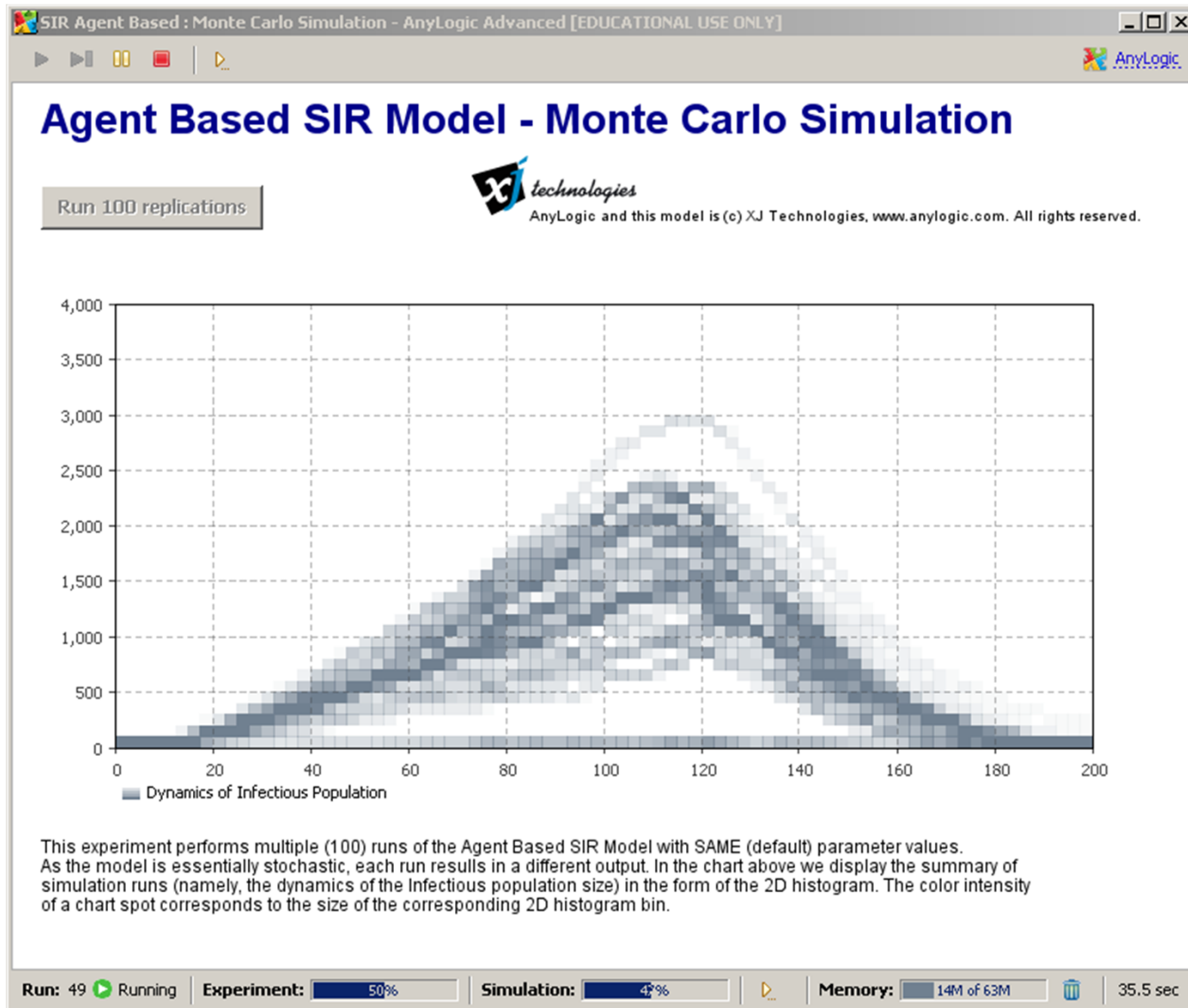
Auto update after every iteration

Monte Carlo Analyses in AnyLogic: Specifying Distributions for Parameters

The screenshot displays the AnyLogic Advanced interface for a Monte Carlo simulation. The main window shows a 2D histogram titled "Agent Based SIR Model - Monte Carlo Simulation" with the y-axis labeled "infectious2D" ranging from 2,000 to 4,000. A button "Run 100 replicat..." is present. The bottom panel, "MonteCarlo2DHistogram - Parameter Variation Experiment", includes a table of parameters:

Parameter	Expression
Average...ration*	max(0,normal(5,15))
ContactRate	1.0
Infection...bability	0.8
AreaSide	100
TotalPopulation	10000

Monte Carlo Output After All Runs



Monte Carlo Output After All Runs

